Undergraduate lectures of Prof. Gleb A. Koshevoy



**Gleb A. Koshevoy**

Professor Gleb A. Koshevoy is a Russian mathematician. He earned his PhD in Moscow State University.

**Professional Experience**

Principal, Leading, Senior, Junior Researcher; Central Economics and Mathematics Institute of the Russian Academy of Sciences

**Research Interests**

Mathematical Economics, Algebraic Combinatorics, Mathematical Statistics, Combinatoric and Discrete Convexity, Algebraic Geometry.

**Homepage**

<http://mathecon.cemi.rssi.ru/en/koshevoy/index.htm>

The Lectures for undergraduate has three part as following,

* + 3 lectures: Combinatorics of Young tableaux and arrays
  + 3 lectures: Combinatorics of polytopes
  + 4 lectures: Crystal bases and local characterization of Kashiwara crystals

| **Mon** | **Tue** | **Wed** | **Thu** | **Fri** | **Sat** | **Sun** |
| --- | --- | --- | --- | --- | --- | --- |
|  | 11.21  18:30-20:30 |  | 11.23  18:30-20:30 | 11.24  18:30-20:30 |  |  |
|  | 11.28  18:30-20:30 |  | 11.30  18:30-20:30 |  |  |  |

1. Lectures for undergraduate

Address: Online

Time:

1. 23.11.21 (Tue.) 18:30-20:30
2. 23.11.23 (Thu.) 18:30-20:30
3. 23.11.24 (Fri.) 18:30-20:30
4. 23.11.28 (Tue.) 18:30-20:30
5. 23.11.30 (Thu.) 18:30-20:30

Teaching assistant: 张卢俊Zhang Lujun

(Ph.D. student, 浙江大学学号Zhejiang University ID 12135007)

Here is a plan of lectures for undergraduate students:

**Topic1. Combinatorics of Young tableaux and arrays**

This topic (3 lectures) is about following two articles:

* **The octahedron recurrence and RSK-correspondence**

<https://arxiv.org/abs/math/0703414>

Abstract

WemakethestatementrigorousthattheRobinson–Schensted–Knuthcorrespondence is a tropicalization of the Dodgson condensation rule.In the present paper the authors use a tropicalization of Dodgson's condensation rule to recover their reformulated RSK correspondence, thus making rigorous the claim that the RSK correspondence is a tropicalization of Dodgson's condensation rule.

* **Arrays and the octahedron recurrence**

<https://arxiv.org/abs/math/0504299>

Abstract

Recently, in papers by Knutson, Tao and Woodward, Henriques and Kamnitzer, Pak and Vallejo have been constructed several interesting bijections of associativity and commutativity. In the first two papers bijections relate special sets of discretely concave functions (hives) on triangular grids and the octahedron recurrence plays the key role for these bijections. Pak and Vallejo related special sets of Young tableaux and constructions of these bijections based on standard algorithms in this theory, jeu de taquen, Schutzenberger involution, tableaux switching, etc. In this paper we investigate these constructions from the third point of view, combinatorics of arrays, theory worked out recently by the authors. Arrays naturally related as well to functions on the lattice of integers as to Young tableaux. In the tensor category of arrays, the bijections of associativity and commutativity arise naturally. We establish coincidence of our bijections with that defined in the first two papers and in the integer-valued set-up with the bijection in the third paper (that is, in particular, a solution of Conjecture 1 by Pak and Vallejo). In order to relate different approaches and to reveal combinatorics of the octahedron recurrence, we, first, show that the octahedron recurrence agrees with discrete convexity and, second, we construct another bijection using the octahedron recurrence, the functional form of the RSK correspondence.

**Topic 2. Combinatorics of polytopes**

This topic (3lectures) is about following objects:

* flow polytopes
* MV-polytopes
* polymatroids
* positroids

**Topic 3. Crystal bases and local characterization of Kashiwara crystals**

This topic (4 lectures) is about following three articles:

* **On the combinatorial structure of crystals of types A,B,C**

<https://arxiv.org/abs/1201.4549>

Abstract

Regular A\_n-, B\_n- and C\_n-crystals are edge-colored directed graphs, with ordered colors 1, 2, . . . , n, which are related to representations of quantized algebras U\_q(sl\_{n+1}), U\_q(sp\_{2n}) and U\_q(so\_{2n+1}), respectively. We develop combinatorial methods to reveal refined structural properties of such objects.

Firstly, we study subcrystals of a regular A\_n-crystal K and characterize pairwise intersections of maximal subcrystals with colors 1, . . . , n-1 and colors 2, . . . , n. This leads to a recursive description of the structure of K and provides an efficient procedure of assembling K.

Secondly, using merely combinatorial means, we demonstrate a relationship between regular B\_n-crystals (resp. C\_n-crystals) and regular symmetric A\_{2n-1}- crystals (resp. A\_{2n}-crystals).

* **B\_2-crystal: axioms, structure, models**

<https://arxiv.org/abs/0708.2198> , <https://arxiv.org/abs/1912.12697>

Abstract

We present a list of `local' axioms and an explicit combinatorial construction for the regularB2-crystals (crystal graphs of irreducible highest weight integrable modules overU\_q(sp4)). Also, a new combinatorial model for these crystals is developed.

In this erratum we explain how to implement some axioms stated in our paper of 2009 so as to obtain a purely "local" characterization for finiteB\_2-crystals, which was declared but not clarified at some moments there. Also we correct some inaccuracies in that paper.

* **Assembling crystals of type A**

<https://arxiv.org/abs/1212.5771>

Abstract

Regular A\_n-crystals are certain edge-colored directed graphs which are related to representations of the quantized universal enveloping algebra Uq(sln+1). For such a crystal K with colors 1,2,...,n, we consider its maximal connected subcrystals with colors 1,...,n − 1 and with colors 2,...,n and characterize the interlacing structure for all pairs of these subcrystals. This is used to give a recursive description of the combinatorial structure of K and develop an efficient procedure of assembling K.